

Euclidean wormholes in string theory

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ABSTRACT: We show that toroidal compactification of type II string theory to six dimensions admits axionic euclidean wormhole solutions. These wormholes can be inserted into $AdS_3 \times S^3 \times T^4$ backgrounds, which have a well-defined CFT dual. AdS/CFT duality then suggests that the wormhole solutions cannot be interpreted using α parameters as originally suggested by Coleman.

KEYWORDS: AdS-CFT Correspondence, Gauge-gravity correspondence.

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1. Introduction

Euclidean wormholes [1–3] are extrema of the action in euclidean quantum gravity, connecting two different asymptotic regions, or arbitrarily separated points in the same geometry. These are clearly interesting objects. The former configurations might provide for some imprint of other vacua in the landscape on physics in our vacuum. The latter configurations induce in the effective action (on scales larger than the wormhole size) bilocal operators of the form

$$S_{WH} = -\frac{1}{2} \sum_{IJ} \int d^D x d^D y \mathcal{O}_I(x) C_{IJ} \mathcal{O}_J(y), \tag{1.1}$$

where the $\mathcal{O}_I(x)$ are local operators with the same quantum numbers as the mouth of the wormhole at x . Naively these operators completely destroy macroscopic locality. However, Coleman argued for a different interpretation [4, 5] noting that

$$e^{-S_{WH}} = \int d\alpha_I e^{-\frac{1}{2} \alpha_I (C^{-1})_{IJ} \alpha_J} e^{-\int d^D x \sum_I \alpha_I \mathcal{O}_I(x)}. \tag{1.2}$$

Thus, all correlation functions are those of a local theory with the addition of the operators $\alpha_I \mathcal{O}_I$ to the Lagrangian density, and with a Gaussian weighting for the α_I . Note that the α_I are constant in space and time. Branches of the wavefunctions with different values of the α_I will decohere, and a given universe can be thought of as being in a superselection sector labeled by a specific set of these parameters.

If wormholes exist in quantum gravity and are to be interpreted a la Coleman, then they represent an intrinsic randomness of the observed constants of nature, even if the

fundamental Lagrangian is completely fixed and we are in a specified vacuum.¹ Thus it is important to understand whether this effect is present. AdS/CFT duality should provide a laboratory for addressing this question within string theory, as it represents a complete description of quantum gravity within AdS boundary conditions. There have been some studies of this question [7, 8] but as yet no sharp result, because the wormholes that have been considered have fields that grow at long distance on at least one side. Thus they cannot be inserted into AdS space without changing the boundary conditions, and their effects cannot be represented locally on each end as in eq. (1.1).

It is clearly of interest to try and embed wormholes into string theory and in particular into AdS/CFT. The simplest controllable wormhole solutions, with parametrically low curvature relative to the Planck scale, are the axionic wormholes of Giddings and Strominger [1]. These have yet to be embedded in string theory, for the usual reason that axions are always accompanied by dilatonic moduli which must be taken into account. The simplest attempt at doing so leads to singular solutions [9]. Subsequent work by Tamvakis [10] and much more recent work of Bergshoeff et. al [11] revealed wormholes solutions in string compactifications down to $D = 4$ flat spacetime. These solutions can be embedded into euclidean $AdS_2 \times S^2$ by arranging for the wormhole size to be much less than the AdS curvature. However, this does not yet allow us to pose a sharp paradox, because the CFT_1 duals remain mysterious and so it is hard to draw conclusions. Another strategy would be to use the recent compactifications of type IIA supergravity down to AdS_4 with all moduli except some axions stabilized [12]. However, we again do not understand the CFT duals of these models; also, it is not clear whether the moduli are sufficiently massive.

The above results are consistent with the possibility that the moduli singularities of string wormholes always conspire to prevent a sharp confrontation with AdS/CFT duality [7].² However, we will find that this is not the case.

In section 2 we review and generalize the constructions of wormholes in flat spacetime of refs. [1, 9–11]. For euclidean gravity coupled to scalar moduli with a general metric on moduli space, we show that non-singular wormhole solutions exist when there are sufficiently long timelike geodesics in moduli space, measured in Planck units. We then check whether the simplest moduli spaces that arise from toroidal compactification down to D flat dimensions satisfy this constraint. For a simple class of geodesic trajectories, we find that while there are no solutions for $6 < D \leq 10$, in $D \leq 6$ non-singular wormhole solutions exist. The $D = 6$ solutions can be embedded (again on scales smaller than the AdS curvature) into $AdS_3 \times S^3 \times T^4$, where there is a well-defined CFT dual.

In section 3 we discuss some technical subtleties related to the matter path integral. This is not central to our main point, but is necessary in particular to resolve a puzzle regarding the single-valuedness of the fields. We also evaluate the wormhole action, taking

¹Coleman later proposed an additional mechanism that would fix the constants [6], and which would in particular set the cosmological constant to zero. This requires additional assumptions about the interpretation of euclidean gravity, and has been criticized on various grounds.

²There is a fairly large literature on string wormhole solutions whose metric is nonsingular in some frame, but where the dilaton diverges at one end so these cannot be glued into a single spacetime. Ref. [13] discusses wormholes that are asymptotically nonsingular but have a singularity in the middle.

account of surface terms that are ignored in the earlier discussion.

In section 4 we discuss the implications of our result. We draw a sharp paradox between AdS/CFT duality and the fluctuation of the coupling constants, and so argue that Coleman’s effect must not be present in string theory. We also discuss alternate interpretations.

2. Wormhole solutions

2.1 Generalities

The simplest setting for string wormhole solutions [1, 9] is a euclidean theory in D dimensions, with gravity, a scalar, and a $(D - 1)$ -form field strength. The action is

$$S_F = \frac{1}{2\kappa^2} \int d^D x \sqrt{g} \left(-\mathcal{R} + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{2(D-1)!} e^{\beta\varphi} F_{\mu\dots\nu} F^{\mu\dots\nu} \right), \quad (2.1)$$

up to a surface term that we leave for the next section. The equations of motion for this action are the same as for

$$S_A = \frac{1}{2\kappa^2} \int d^D x \sqrt{g} \left(-\mathcal{R} + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} e^{-\beta\varphi} \partial_\mu A \partial^\mu A \right), \quad (2.2)$$

with the form field replaced by an ‘axion’ via

$$F_{\mu\dots\nu} = \epsilon_{\mu\dots\nu\lambda} e^{-\beta\varphi} \partial^\lambda A. \quad (2.3)$$

We will refer to eqs. (2.1), (2.2) as the flux form and the axion form respectively. Note that the Lagrangian density is not invariant, and that for real $F_{\mu\dots\nu}$ the A kinetic term is negative. For now we use the scalar version simply as a means of encoding the equations of motion. Subtleties such as the single-valuedness of A , and all surface terms in the action, will be left for the next section.

We now generalize to an arbitrary euclidean theory of gravity coupled to massless scalars ϕ_I in D dimensions. The two-derivative action is

$$S_A = \frac{1}{2\kappa^2} \int d^D x \sqrt{g} \left(-\mathcal{R} + \frac{1}{2} G_{IJ}(\phi) \partial_\mu \phi_I \partial^\mu \phi_J \right). \quad (2.4)$$

As we see in the example above, for the euclidean Lagrangians coming from compactifications of string theory, the metric G_{IJ} does not have a definite signature — in particular the axionic scalars have the “wrong” sign kinetic terms. Indeed these wrong signs are crucial for the wormhole solutions to exist.

We are interested in spherically symmetric solutions of the form

$$ds^2 = N^2(r) dr^2 + a^2(r) d\Omega_{D-1}^2, \quad \phi_I = \phi_I(r). \quad (2.5)$$

Plugging this Ansatz into the action we have

$$S_A = \frac{V_{D-1}}{2\kappa^2} \int dr N a^{D-1} \left[(D-1)(D-2) \left(-\frac{a'^2}{N^2 a^2} - \frac{1}{a^2} \right) + \frac{1}{2N^2} G_{IJ} \phi^{I'} \phi^{J'} \right], \quad (2.6)$$

where primes will always denote derivatives with respect to r . Here we have neglected surface terms that don't affect the equations of motion, we will take them into account in evaluating the action in the next section.

Varying N , and then choosing the gauge $N(r) = 1$, gives the constraint

$$\frac{a'^2}{a^2} - \frac{1}{a^2} - \frac{G_{IJ}\phi^{I'}\phi^{J'}}{2(D-1)(D-2)} = 0. \quad (2.7)$$

The equation of motion for the scalars is

$$(a^{D-1}G_{IJ}\phi^{J'})' - \frac{1}{2}a^{D-1}G_{JK,I}\phi^{J'}\phi^{K'} = 0. \quad (2.8)$$

If we define $dr/a^{D-1} = d\tau$, this becomes the geodesic equation in the metric G_{IJ} ,

$$\frac{d^2\phi^I}{d\tau^2} + \Gamma^I_{JK}\frac{d\phi^J}{d\tau}\frac{d\phi^K}{d\tau} = 0. \quad (2.9)$$

In particular these means that the invariant length $G_{IJ}(d\phi^I/d\tau)(d\phi^J/d\tau)$ is a constant of the motion, or equivalently that

$$G_{IJ}\phi^{I'}\phi^{J'} = \frac{C}{a^{2D-2}}. \quad (2.10)$$

Inserting this into the equation for a' , we have

$$a'^2 - 1 - \frac{C}{2(D-1)(D-2)a^{2D-4}} = 0, \quad (2.11)$$

which is the same as energy conservation for a newtonian particle with effective potential

$$V_{\text{eff}}(a) = -1 - \frac{C}{2(D-1)(D-2)a^{2D-4}}. \quad (2.12)$$

The nature of the solution then depends on whether the geodesic motion on moduli space is spacelike ($C > 0$), null ($C = 0$) or timelike ($C < 0$). For $C > 0$, the potential goes to $-\infty$ as $a \rightarrow 0$, so a' must diverge and the solution is singular. For $C = 0$, we have $a(r) = r$, so the metric is that of flat space and the scalar solution corresponds to an extremal D-instanton. But clearly for

$$C \equiv -2(D-1)(D-2)a_0^{2D-4} < 0, \quad (2.13)$$

wormhole solutions are possible with $a(r) \rightarrow r^2$ as $r \rightarrow \pm\infty$, and with minimum value $a(r=0) = a_0$.

We see that the scalars are just travelling along a timelike geodesic in moduli space, but there is a constraint in order to be able to find a solution. Denote the value of the moduli at the spacetime boundaries as $\phi_{\pm\infty}$ and the value at the neck by ϕ_0 . Now, the timelike distance between $\phi_{-\infty}$ and ϕ_{∞} along the geodesic is

$$\begin{aligned} d[\phi_{-\infty}, \phi_{\infty}] &= 2d[\phi_{-\infty}, \phi_0] = 2 \int_{-\infty}^0 dr \frac{|C|^{1/2}}{a(r)^{D-1}} \\ &= 2\sqrt{2(D-1)(D-2)} \int_1^{\infty} \frac{d\hat{a}}{\hat{a}\sqrt{\hat{a}^{2D-4} - 1}} \\ &= \pi\sqrt{\frac{2(D-1)}{D-2}}, \end{aligned} \quad (2.14)$$

where we substituted $dr \rightarrow da/a'(r)$, used the equation of motion for a' , and introduced the dimensionless variable $\hat{a} = a/a_0$. Thus, in order to be able to find a wormhole solution with the moduli bounded, we must be able to find a compact timelike geodesic at least as long as (2.14) in the scalar moduli space. In practice we will identify noncompact timelike geodesics, whose length must be strictly greater than (2.14),

$$d^2 > 2\pi^2 \frac{D-1}{D-2}, \tag{2.15}$$

so that we can restrict to a compact piece satisfying (2.14). This is measured in Planck units, $2\kappa^2 = 1$.

It is trivial to generalize this analysis to a case with negative cosmological constant, so that the asymptotic spaces are AdS spaces with curvature scale L . This is just an aside; it is not directly relevant to the case we will be interested in, instantons localized on AdS×S spaces. The cosmological constant simply adds a piece $-a^2/L^2$ to the effective potential $V(a)$, and the solutions asymptote to $a(r) \rightarrow e^{|r|/L}$ as $r \rightarrow \infty$. Following the same steps as before, the length of the geodesic is becomes

$$d[\phi_{-\infty}, \phi_0] = \sqrt{2(D-1)(D-2)} \int_1^\infty \frac{d\hat{a}}{\hat{a} \sqrt{\hat{a}^{2D-4}(\hat{a}^2 + \hat{L}^2)/(1 + \hat{L}^2) - 1}}, \tag{2.16}$$

where $\hat{a} = a/a_0$, $\hat{L} = L/a_0$, and a_0 is again the turning radius. This is strictly less than the flat space integral, so the necessary geodesic is shorter (a weaker condition). Clearly for small wormholes relative to the AdS scale $a_0/L \ll 1$, the bound on the length is the same as before up to corrections of order $O(a_0^2/L^2)$.

The condition of having a long enough timelike geodesic in moduli space is trivially satisfied for the axion-gravity system without a dilaton [1], where the moduli space is one-dimensional and the metric $ds^2 = -dA^2$ is timelike thanks to the “wrong” sign of the axion kinetic term. However the moduli spaces we get from simple compactifications of string theory have both space-like and time-like directions, associated with axions and dilatons. In particular, these Lorentzian moduli spaces have horizons that limit the length of timelike geodesics. For instance, consider the axion-dilaton system in type IIB string theory in $D = 10$. The moduli space metric (with the wrong sign for the axion) is

$$ds^2 = d\varphi^2 - e^{2\varphi} dA^2, \tag{2.17}$$

corresponding to $\beta = -2$ in the action (2.1), (2.2). This is just minus the metric of a causal patch of $(1 + 1)$ -dimensional de Sitter space with unit dS radius, so we are interested in spatial geodesics in this patch. As is familiar, because of the presence of the dS horizon there is a maximum separation between spacelike separated points beyond which no connect can connect them, so there is a maximum length for spatial geodesics.

This can be found by an easy direct computation: there is a noncompact timelike geodesic $e^\varphi = \cos \tau$, $A = \tan \tau$, whose length is π . We can also get the answer indirectly, by remembering that the Wick rotation of a causal patch of dS is half of a ball S^2 , so the

maximum separation angle, and the maximum length on the unit hemisphere, is π .³ There is no D for which this exceeds $\pi\sqrt{\frac{D-1}{D-2}}$ (the case $D = 1$ is unphysical), so there are no wormhole solutions.

For general β , the metric can be written

$$ds^2 = d\varphi^2 - e^{-\beta\varphi}dA^2 \equiv \frac{4}{\beta^2}(d\tilde{\varphi}^2 - e^{2\tilde{\varphi}}d\tilde{A}^2). \quad (2.18)$$

The longest geodesic is now $2\pi/|\beta|$, and so the condition (2.15) for a wormhole solution becomes [1, 9, 14]

$$\frac{1}{\beta^2} > \frac{D-1}{2(D-2)}. \quad (2.19)$$

For reference we give the most general long geodesic for given β , whose four parameters $\varphi_0, A_0, \tau_0, q$ can be obtained by a general $SL(2, R)$ transformation together with a rescaling of the affine parameter, which corresponds to scaling the wormhole charge (a shift of the affine parameter has the same effect as one of the $SL(2, R)$ generators):

$$e^{-\beta\varphi/2} = e^{-\beta\varphi_0/2} \cos q(\tau - \tau_0), \quad A = A_0 - \frac{2}{\beta} e^{\beta\varphi_0/2} \tan q(\tau - \tau_0), \quad (2.20)$$

The length, for $-\pi/2 < q\tau < \pi/2$, is always $2\pi/|\beta|$.

2.2 Wormholes in $D = 4, 6$

For heterotic or type II superstring theory compactified to $D = 4$ on a Calabi-Yau manifold or T^6 , the effective action for the four-dimensional dilaton and axion $(\Phi_4, B_{\mu\nu})$ plus the internal dilaton and axion (σ, A) is

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{g} \left(-\mathcal{R} + 2\partial_\mu \Phi_4 \partial^\mu \Phi_4 + 6\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{12} e^{-4\Phi_4} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{2} e^{-4\sigma} \partial_\mu A \partial^\mu A \right). \quad (2.21)$$

Upon rescaling the fields, this is two copies of the system (2.1), (2.2), the four-dimensional one having $\beta^{-2} = \frac{1}{4}$ and the internal one having $\beta^{-2} = \frac{3}{4}$ [9]. The condition (2.19) becomes $\beta^{-2} > \frac{3}{4}$ so both of these wormholes are singular.⁴ However, a simple observation [10, 11] allows the construction of nonsingular wormholes in this theory. That is, if we consider a solution with *both* axion charges, then the relevant moduli space is the product of the two separate spaces, and the longest timelike geodesic would be the ‘diagonal’ in the two spaces. The condition (2.19) now becomes

$$\sum_i \frac{1}{\beta_i^2} > \frac{D-1}{2(D-2)}, \quad (2.22)$$

³There is a geodesic of length 2π running around the edge, but there is no slicing such that this continues back to a Lorentzian geodesic.

⁴For the critical case $\beta^{-2} = \frac{3}{4}$, the fields blow up only at infinity, and it is possible that an extension of the analysis (2.16) to $AdS_2 \times S^2$ would give a nonsingular wormhole. However, to make the sharpest paradox we would like to be able to take the wormholes small compared to the AdS radius, so we insist that they be nonsingular even in flat space.

which is comfortably satisfied in the theory (2.21). Thus euclidean wormhole solutions do exist in string theory [10, 11].

Turning to the IIB theory on T^4 , for which we have a good CFT dual, the most closely analogous action would involve the four-dimensional and internal dilatons, as well as the noncompact RR five-form field strength and internal axion. The reduced action is

$$S = \frac{1}{2\kappa^2} \int d^6x \sqrt{g} \left(-\mathcal{R} + \partial_\mu \Phi_6 \partial^\mu \Phi_6 + 4\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{12} e^{-2\Phi_6 + 4\sigma} F_{\mu\dots\rho} F^{\mu\dots\rho} - \frac{1}{2} e^{-4\sigma} \partial_\mu A \partial^\mu A \right). \quad (2.23)$$

The internal dilaton-axion system has $\beta^{-2} = \frac{1}{2}$. Exciting the five-form sources the linear combination $\Phi_4 = -2\sigma$, giving $\beta^{-2} = \frac{1}{4}$. Neither of these exceeds the necessary value $\frac{5}{8}$. Also, in this case we cannot simply combine the two systems diagonally as in eq. (2.22) because the moduli space is not a product: the dilatons mix, and so do the axions (through Chern-Simons couplings); we do not know if there is a sufficiently long geodesic in this space.

However, a simple trick allows us to find nonsingular wormholes in a different way. Consider just the internal fields, and regard the T^4 as $T^2 \times T^2$. We now have a product space where each piece has $\beta^{-2} = \frac{1}{4}$, so that the diagonal geodesic has length-squared $\frac{1}{4} + \frac{1}{4}$, reproducing the result for the internal dilaton-axion of T^4 . Now, it is familiar that for compactification on T^2 we can identify *two* decoupled dilaton-axion systems, where the first is from the dilaton-axion on the T^2 and second comes from the complex structure of the T^2 . A Z_2 T -duality interchanges these so they must each have $\beta^{-2} = \frac{1}{4}$. Summing over the four separate factors from the two T^2 s, the left-hand-side of eq. (2.22) is $1 > \frac{5}{8}$, and so there are nonsingular wormholes with six noncompact dimensions. Thus we will be able to frame a sharp paradox with AdS/CFT.

The axions for the solution just described come from g_{67} , B_{67} , g_{89} , and B_{89} . We can also construct this solution in various dual forms. For example, by taking the S -dual, and then the T -dual on the 7- and 9-axes, we obtain instead the axions B_{67} , C_{69} , B_{89} , and C_{78} . The reduced action is

$$S = \frac{1}{2\kappa^2} \int d^6x \sqrt{g} \left(-\mathcal{R} + \partial_\mu \Phi_6 \partial^\mu \Phi_6 + \sum_{m=6}^9 \partial_\mu \sigma_m \partial^\mu \sigma_m - \frac{1}{2} e^{-2\sigma_6 - 2\sigma_7} \partial_\mu B_{67} \partial^\mu B_{67} - \frac{1}{2} e^{-2\sigma_8 - 2\sigma_9} \partial_\mu B_{89} \partial^\mu B_{89} - \frac{1}{2} e^{2\Phi_6 - \sigma_6 + \sigma_7 + \sigma_8 - \sigma_9} \partial_\mu C_{69} \partial^\mu C_{69} - \frac{1}{2} e^{2\Phi_6 + \sigma_6 - \sigma_7 - \sigma_8 + \sigma_9} \partial_\mu C_{78} \partial^\mu C_{78} \right) \quad (2.24)$$

Note that the axions couple to orthogonal combinations of moduli, and that the normalization corresponds to $\beta^{-2} = \frac{1}{4}$ for each. For concreteness, we will focus on this example in the following discussion.

The $D = 6$ wormhole solution can also be understood in terms of the $SO(5, 5)/SO(5) \times SO(5)$ geometry of the moduli space. For the numerator group we have $SO(5, 5) \supset SO(4, 4) \supset SO(2, 2)^2 = SO(2, 1)^4$; the last step is parallel to the familiar $SO(4) = SO(3) \times SO(3)$. The intersection of the denominator group with the $SO(2, 1)^4$ is $SO(2)^4$. Thus we obtain four copies of the dilaton-axion system. The construction with axions g_{67} , B_{67} ,

g_{89} , and B_{89} corresponds to $SO(4, 4)$ and $SO(2, 2) \times SO(2, 2)$ being the real versions of the T -duality groups of T^4 and $T^2 \times T^2$ respectively. The axions B_{67} , C_{69} , B_{89} , and C_{78} lie in a U -dual $SO(4, 4)$. In $D = 7$ the numerator group is $SL(5, R)$ and only contains two copies of $SO(2, 1)$, which is not enough for our construction. It is possible that there are longer geodesics not lying in a product of $SO(2, 1)/SO(2)$ factors, but we have not been able to find any.

3. Technicalities

3.1 Path integral subtleties

We begin with a discussion of the structure of the euclidean action, in particular the peculiar “wrong” sign kinetic terms in the axion form of the action compared to the “normal” sign kinetic term for the “flux” form. From a variety of perspectives, it is nice to understand what is going on in axionic language; for instance because winding D and F strings have a natural local coupling to axions. Much of the discussion below is a review of the arguments of Coleman and Lee [15]. We then compute the action for the wormhole solutions found in the previous section. Interestingly, we find that this action is always *smaller* than that of a pair of D-instantons with the same charges at the two mouths of the wormhole.

The relevant issues all arise in a very simple and familiar toy setting: the quantum mechanics of a non-relativistic particle moving on a circle of unit radius, with euclidean action $S = \int dt \left(\frac{1}{2} \dot{\theta}^2 + V(\theta) \right)$. Here θ is the analog of our axions. With vanishing potential, there is shift symmetry on θ with associated charge Q — the particle momentum. We include the possibility of a small $V(\theta)$ to model the tiny shift-symmetry violating effects we also expect in our axion example. Now consider the euclidean transition amplitude

$$G_{\theta_f, \theta_i}(T) = \langle \theta_f | e^{-HT} | \theta_i \rangle \tag{3.1}$$

Let us start with $V = 0$. In this trivial case we know the spectrum exactly and

$$G_{\theta_f, \theta_i}(T) = \sum_Q e^{-Q^2 T/2} e^{iQ(\theta_f - \theta_i)} \tag{3.2}$$

Clearly, for large T , this expression has the form of a semiclassical expansion, with increasingly exponentially suppressed contributions from larger charge sectors; indeed this sum is the direct analog of the “flux” presentation of the action. On the other hand, a straightforward evaluation of the path integral representation of $G_{\theta_f, \theta_i}(T)$ does not yield this semiclassical expansion. Indeed, the saddle points of the euclidean path integral with paths starting at θ_i and ending at θ_f in euclidean time T are ones that wind around the circle m times, so $\theta(\tau) = \theta_i + (\theta_f - \theta_i + 2\pi m)\tau/T$, with euclidean action $(\theta_f - \theta_i + 2\pi m)^2/2T$. Thus

$$G_{\theta_f, \theta_i}(T) = \sum_m e^{-(\theta_f - \theta_i + 2\pi m)^2/2T} \tag{3.3}$$

Of course this expression for G is the same as the earlier one by Poisson resummation (modulo a prefactor from the determinant, which we have omitted), but for large T the

direct evaluation of the euclidean path integral gives a very bad expansion. It would be nice to extract the good semiclassical expansion directly from a θ path integral. To wit, let us look at the euclidean transition amplitude, not between eigenstates of θ but between momentum eigenstates:

$$G_{Q',Q}(T) = \langle Q' | e^{-HT} | Q \rangle, \quad G_{\theta_f, \theta_i}(T) = \sum_{Q', Q} e^{i(Q'\theta_f - Q\theta_i)} G_{Q', Q}(T) \quad (3.4)$$

Clearly for $V(\theta) = 0$, charge is conserved and $G_{Q', Q}$ is diagonal, while for small $V(\theta)$ there will be small off-diagonal pieces. Now, since $|Q\rangle = \int d\theta e^{iQ\theta} |\theta\rangle$, there is a simple path integral representation for $G_{Q', Q}(T)$:

$$G_{Q', Q}(T) = \int_{\text{free}} \mathcal{D}\theta e^{-\tilde{S}(\theta)}, \quad \tilde{S}(\theta) = S(\theta) - i[Q\theta(0) - Q'\theta(T)], \quad (3.5)$$

where the first term is the usual euclidean action $S(\theta) = \int_0^T \left(\frac{1}{2} \dot{\theta}^2 + V(\theta) \right)$ and the second term is a boundary action. Note that the boundary values of θ are not fixed but free (integrated over) in this path integral. To belabor the obvious — if we are interested in G_{θ_f, θ_i} , the boundary values θ_f, θ_i enter only in the fourier transform from $G_{Q', Q}$ to G_{θ_f, θ_i} , and have *nothing* to do with the (unfixed!) boundary values in the θ path integral.

Now, the saddle points contributing to the θ path integral are easily determined. The equation of motion for θ is the usual euclidean one $\ddot{\theta} - V'(\theta) = 0$, but there is also a boundary variation which leads to

$$\dot{\theta}(T) = -iQ', \quad \dot{\theta}(0) = -iQ. \quad (3.6)$$

Clearly in general these equations have complex solutions. For the special case $V(\theta) = 0$, as expected there is only a solution for $Q = Q'$ which is $\dot{\theta} = iQ$. Alternately, we could define $\theta = -iA$; then A would have the “wrong” sign kinetic term but a real solution. Either way, on the solution, the action is $\tilde{S} = +Q^2 T/2$, and reproduces the nice semiclassical expansion for large T .

We can consider a more interesting example, a particle moving in a central potential in two dimensions with euclidean action $S = \int dt \left(\frac{1}{2} \dot{r}^2 + \frac{1}{2} r^2 \dot{\theta}^2 + V(r) \right)$. There is still a charge associated with the shift symmetry on θ , the angular momentum, and we can still find a semiclassical expansion along the lines above. Clearly while θ in this case will be imaginary, r will not be; equivalently, we flip the kinetic term for θ and not r in evaluating the saddle point action with fixed angular momentum. In both cases, the kinetic term of the variable conjugate to the conserved charge is flipped; note that the action is positive, however, and can be obtained by inserting the solution of the equations of motion with the wrong sign kinetic term back into the original euclidean action with usual sign kinetic terms.

This story generalizes trivially to our wormhole example. In all the cases of interest in string theory, there is a set of co-ordinates on moduli space where the moduli naturally group themselves into dilatons φ_a and axions A_i , with metric $G_{IJ} d\phi_I d\phi_J = G_{ab}(\varphi) d\varphi_a d\varphi_b + \sum_i F_i(\varphi) dA_i^2$. There are charges Q_i associated with the shift symmetries

on the A_i . The wormhole solutions correspond to euclidean transition amplitudes with fixed Q_i , and can be obtained from an action where the kinetic terms for the A_i are flipped, yielding the metric G_{IJ} we considered in the previous section. The wormhole solution follows from varying the action

$$S = \frac{1}{2\kappa^2} \int d^D x \sqrt{g} \left(-\mathcal{R} + \frac{1}{2} G_{IJ} \partial_\mu \phi_I \partial^\mu \phi_J \right) \quad (3.7)$$

In order to get a good semiclassical expansion, we must insert projections onto definite Q at the wormhole ends, as above, and then on equations of motion the semiclassical action is given by \tilde{S} , with the signs of the axion kinetic terms flipped. For transitions between asymptotic regions at points (φ_a, A_i) and (φ'_a, A'_i) in the moduli space, the wormhole solution need only interpolate between φ_a, φ'_a , with no regard for the dependence on A_i, A'_i — the latter are discontinuous at the projection operators.

In order for this semiclassical approximation to be valid, the wormhole should have large action. The wormhole carries the same charges as wrapped F- and D-string instantons, so we need

$$L^2/\alpha' \gg 1, \quad L^2/\alpha' g_s \gg 1, \quad (3.8)$$

where L is the size of the T^4 , assumed to be roughly isotropic. Winding F- and D-string states couple to the axions. In the regime (3.8), these are massive compared to other scales and can be integrated out, giving rise to small breaking of the shift symmetry on the A_i .⁵ Also, in order for the supergravity description of the wormhole to hold, the throat must be large in string units. This implies at least one of Q_{Rg_s} and $Q_{NSg_s^2}$ must be large, where $Q_{R,NS}$ are the charges flowing through the wormhole throat.

3.2 The wormhole action

In our toy example, the action S corresponds to the axion action S_A , eq. (2.2), while the additional surface term in \tilde{S} is equivalent, upon use of the equation of motion, to flipping the sign of the axion action as in S_F , eq. (2.1). Following the previous discussion, we use the latter in evaluating the semiclassical action:

$$S_F = S_A + \frac{1}{2\kappa^2} \int d^D x \sqrt{g} \sum_i F_i(\varphi) \partial_\mu A_i \partial^\mu A_i, \quad S_A = \frac{1}{2\kappa^2} \int d^D x \sqrt{g} \left(-\mathcal{R} + \frac{1}{2} G_{IJ} \partial \phi_I \partial \phi_J \right). \quad (3.9)$$

In addition, the gravitational action requires a surface term involving the extrinsic curvature of the boundary minus the extrinsic curvature of the boundary embedded in flat spacetime [16]. However, this vanishes for euclidean wormhole solutions, because these approach flat spacetime sufficiently rapidly at infinity [1].

⁵R. Myers points out that due to the complex value for the axion field in the wormhole solution, the masses-squared of these winding states may sometimes acquire a *negative* real part. This should not have any effect, since it should make sense to integrate out the winding states first and then continue to complex A_i . One could investigate this by adding a massive winding state field to the particle model above - it cannot have a large effect on the amplitude.

Now, very generally S_A vanishes on equations of motion, since the trace of the Einstein equation immediately implies $\mathcal{R} = \frac{1}{2}G_{IJ}\partial\phi_I\partial\phi_J$. Using our Ansatz for the metric and scalar solution, the second term and hence the wormhole action is

$$\frac{2\kappa^2}{V_{D-1}}S_F(Q_i) = \int dr a^{D-1} \sum_i F_i(\varphi) A_i'^2 = \int dr Q_i A_i' = \sum_i Q_i \Delta A_i . \quad (3.10)$$

Here $Q_i = a^{D-1} F_i(\varphi) A_i'$ is the i 'th conserved charge, and ΔA_i are the changes from one end of the wormhole to the other. For the solution (2.20),

$$Q_i = -\frac{2}{\beta_i} e^{-\beta_i \varphi_{i0}/2} q_i ; \quad \Delta A_i = -\frac{4}{\beta_i} e^{\beta_i \varphi_{i0}/2} \tan q_i \tau_\infty = -\frac{4}{\beta_i} e^{\beta_i \varphi_{i\infty}/2} \sin q_i \tau_\infty , \quad (3.11)$$

For our $D = 6$ solution, each $\beta_i = 2$ and the condition (2.14) becomes

$$\frac{1}{4} \sum_{i=1}^4 \left(\frac{2}{\pi} q_i \tau_\infty \right)^2 = \frac{5}{8} . \quad (3.12)$$

The parameter $q_i \tau_\infty$ must be less than $\pi/2$ for all i in order to have a nonsingular solution; this allows some region of parameter space, corresponding to different ratios the four charges.

Ref. [1] also considers a topological term γ in the action, proportional to the Euler number of the wormhole. This is the analog of the string coupling constant in the worldsheet expansion. For Calabi-Yau compactification such a term would descend from a ten-dimensional Euler number term [1], which conceivably could be present with an arbitrary coefficient. However, our compactification has a toroidal factor, so the ten-dimensional Euler number is zero. A four-dimensional Euler number term might also be produced by string and quantum corrections, but we are assuming that the wormhole throat is large so that such higher derivative corrections are small.

For comparison, let us note that for supersymmetric instantons the geodesic is null [17, 11], so that $dA_i = \pm(2/\beta_i)d(e^{\beta_i \varphi_i/2})$. Thus, $\Delta A_i = \pm(2/\beta_i)\Delta(e^{\beta_i \varphi_i/2})$, where we are now referring to the change between the asymptotic region and the instanton center. In fact, $e^{\beta_i \varphi_i/2}$ vanishes at the core, so we can write for a supersymmetric instanton

$$\frac{2\kappa^2 \mathcal{S}(Q)}{\Omega_{D-1}} = \sum_i \left| \frac{2Q_i}{\beta_i} \right| e^{\beta_i \varphi_{i\infty}/2} . \quad (3.13)$$

Since $|\sin q_i \tau_\infty|$ is less than one, the wormhole action (3.10), (3.11) is strictly less than that of a collection of supersymmetric instantons of the same total charge in the place of the two ends of the throat. This is a curious result: for a particle state it would correspond to violation of the BPS bound, but for an instanton there appears to be no sharp conflict with supersymmetry.

There has been an interesting related observation in ref. [18], that the wormhole would map to an impossible gauge theory configuration, in which $(F - \tilde{F})^2$ would have to be negative. That is, the BPS-violating bulk instanton maps to a BPS-violating boundary instanton, and there the action has positivity properties that forbid this.

4. Discussion

Now let us formulate a sharp paradox. The basic idea is that the ends of a wormhole can be arbitrarily separated in time, so that amplitudes will not satisfy cluster decomposition, whereas the dual gauge theory has local time evolution and so will satisfy cluster decomposition. Ref. [7] gave similar arguments to the effect that AdS/CFT duality is inconsistent with α parameters.

The CFT dual to IIB string theory on $AdS_3 \times S^3 \times T^4$ is given by the infrared limit of the $D = 2$, $\mathcal{N} = 4$, supersymmetric gauge theory with gauge group $SU(Q_1) \times SU(Q_5)$ [19]. If one avoids special points on the moduli space of the T^4 [20], the scalar potential for the gauge theory on a circle increases in all directions, so the spectrum should be discrete.

The size of the T^4 is of order $(Q_1/Q_5)^{1/4}$ in string units, so we need $Q_1 \gg Q_5, Q_5 g_s^2$ in order that the conditions (3.8) for the semiclassical expansion be valid. The radii of the $AdS_3 \times S^3$ are of order $(g_s Q_5)^{1/2}$ in string units, so we need $g_s Q_5 \gg 1$ in order that these radii be large compared to the string scale. If these conditions are satisfied then we can arrange the wormhole charges so that the throat is large compared to the string scale and small compared to the AdS radius. We can take Q_1 and Q_5 to be large but finite, and this is a superrenormalizable theory, so there should be no subtlety in regarding this as an ordinary quantum mechanical system.

For nonsingular wormholes the fields fall off as $1/r^4$ in flat spacetime (like the Coulomb Green's function). At longer distance this will go over to the Coulomb Green's function for $AdS_3 \times S^3$,

$$G(\tau, \theta) = \frac{1}{4(\cosh \tau - \cos \theta)^2}, \tag{4.1}$$

where τ and θ are the distance along AdS_3 and S^3 respectively. This is normalizable at infinity, so these wormholes, if present, would represent effects described by the original CFT rather than a perturbation of the CFT [21, 22].⁶ The wormhole ends interact through their long-range fields. The wormhole solution is thus not exact — its action depends on the separation of the wormhole ends. However, this effect falls off exponentially (4.1). In the usual spirit of dilute gas instanton sums, there is not at an exact saddle point of the action but rather a nearly flat plateau parameterized by the positions of the ends.

To make the cluster decomposition argument, consider the gauge theory on a very long periodic Euclidean time T , with one set of operators \mathcal{O}_1 near $\tau = 0$ and another set \mathcal{O}_2 near $\tau = T/2$. Assuming that the vacuum is unique, we have in the gauge theory that

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \langle 0 | \mathcal{O}_1 | 0 \rangle \langle 0 | \mathcal{O}_2 | 0 \rangle + O(e^{-ET/2}) \tag{4.2}$$

where E is the gap to the first excited state. Possibly in some cases the ground state has a finite degeneracy leading to a finite sum of such terms, but no more than this because of our remarks about the scalar potential. On the other hand, if the bulk physics is described

⁶Depending on which U -dual form of the wormhole solution we use, there may be Chern-Simons couplings of the axions. These give rise to AdS masses, and so the Green's function falls off faster.

by α -parameters as in eq. (1.2), we would have the expression

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \int d\alpha e^{-\frac{1}{2}\alpha_I (C^{-1})^{IJ} \alpha_J} \langle 0 | \mathcal{O}_1 | 0 \rangle_\alpha \langle 0 | \mathcal{O}_2 | 0 \rangle_\alpha + O(e^{-E_\alpha T/2}), \quad (4.3)$$

where the subscript α indicates that quantities are to be calculated using the α -shifted action. This is not equivalent to a product (4.2) or a sum of products.⁷

Thus it appears that quantum gravity as constructed via AdS/CFT duality does not include Euclidean wormholes, or if it does then they do not have the expected effect. It has been suggested to us by several people that one might get a different gravity, with wormholes, by modifying the CFT. For example, introducing nonlocal bilinear interactions (1.1) directly into CFT would destroy cluster decomposition. However, this does not seem plausible to us. Modifications of the CFT correspond to perturbations of the boundary conditions, not the bulk dynamics. An experimentalist in the bulk should be able to distinguish a local modification of the dynamics from effects propagating inward from the boundary (for example, by doing measurements within a Faraday cage).

The wormhole solutions we have found pose a sharp paradox with AdS/CFT and the apparent uniqueness of quantum gravity in maximally supersymmetric backgrounds. One might have hoped that string theory would have avoided such paradoxes by not producing effective field theories allowing wormhole solutions, but that does not appear to be the case. Instead, these saddle points of the euclidean path integral apparently do not contribute to the partition function despite no obvious IR pathologies (beyond the usual ones of euclidean gravity). Of course, there is no reason to expect every saddle point of an integral to contribute — this is already the case even for ordinary integrals (such as the Airy integral), but one is left to wonder what are the rules that determine saddles contribute and which don't. What is pathological about the wormhole solutions?

Perhaps there is simply a rule excising topologically non-trivial configurations like wormholes from the approximate sum over geometries. Or it may be that the fact that the action lies below the BPS action is a clue that these solutions are in a region of field space that is not reached by a proper contour rotation. The observation of ref. [18], discussed at the end of the previous section, is a further argument in this direction.

Another interesting observation is that the wormhole solution traverses a large distance in moduli space, in Planck units. In analogous situations in Minkowski space, it is difficult to set up backgrounds which span super-Planckian ranges in moduli space without generating horizons to shield them. Some have taken such arguments to imply that there is no sense in which we can talk about distant vacua in moduli space as really part of the same theory, though there have been no convincing arguments on this issue either way. Our wormhole solutions provide a setting where a similar question can be asked. Wormholes exist in the long-distance theory *only* when super-Planckian distances are traversed in moduli space. There are no horizons these excursions can hide behind in euclidean space, and the naive interpretation of wormholes makes connecting distant parts of moduli

⁷There is a large amount of supersymmetry in the bulk, and because this is a gauge symmetry it must be respected by the effective operators induced by the wormhole. These will therefore start at some high dimension, but this does not affect the problem of principle with cluster decomposition.

space in the same theory compulsory. It is therefore interesting that the apparently correct interpretation — that wormholes don't contribute after all — also censors this connection.

Finally, it may be that wormholes do contribute to the path integral but that the interpretation in terms of fluctuating couplings is not correct. That is, there may be some question in quantum gravity for which these saddles contribute. See for example ref. [23] for an alternate interpretation.

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References

- [1] S.B. Giddings and A. Strominger, *Axion induced topology change in quantum gravity and string theory*, *Nucl. Phys.* **B 306** (1988) 890.
- [2] G.V. Lavrelashvili, V.A. Rubakov and P.G. Tinyakov, *Disruption of quantum coherence upon a change in spatial topology in quantum gravity*, *JETP Lett.* **46** (1987) 167 [*Pisma Zh. Eksp. Teor. Fiz.* **46** (1987) 134].
- [3] S.W. Hawking, *Quantum coherence down the wormhole*, *Phys. Lett.* **B 195** (1987) 337; *Wormholes in space-time*, *Phys. Rev.* **D 37** (1988) 904.
- [4] S.R. Coleman, *Black holes as red herrings: topological fluctuations and the loss of quantum coherence*, *Nucl. Phys.* **B 307** (1988) 867.
- [5] S.B. Giddings and A. Strominger, *Loss of incoherence and determination of coupling constants in quantum gravity*, *Nucl. Phys.* **B 307** (1988) 854.
- [6] S.R. Coleman, *Why there is nothing rather than something: a theory of the cosmological constant*, *Nucl. Phys.* **B 310** (1988) 643.
- [7] S.-J. Rey, *Holographic principle and topology change in string theory*, *Class. and Quant. Grav.* **16** (1999) L37 [[hep-th/9807241](#)].
- [8] J.M. Maldacena and L. Maoz, *Wormholes in AdS*, *JHEP* **02** (2004) 053 [[hep-th/0401024](#)].
- [9] S.B. Giddings and A. Strominger, *String wormholes*, *Phys. Lett.* **B 230** (1989) 46.
- [10] K. Tamvakis, *Two axion string wormholes*, *Phys. Lett.* **B 233** (1989) 107.
- [11] E. Bergshoeff, A. Collinucci, U. Gran, D. Roest and S. Vandoren, *Non-extremal D-instantons*, *JHEP* **10** (2004) 031 [[hep-th/0406038](#)]; *Non-extremal instantons and wormholes in string theory*, *Fortschr. Phys.* **53** (2005) 990 [[hep-th/0412183](#)].
- [12] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Type IIA moduli stabilization*, *JHEP* **07** (2005) 066 [[hep-th/0505160](#)].
- [13] M.B. Einhorn, *Instanton of type IIB supergravity in ten dimensions*, *Phys. Rev.* **D 66** (2002) 105026 [[hep-th/0201244](#)].
- [14] M. Gutperle and W. Sabra, *Instantons and wormholes in Minkowski and (A)dS spaces*, *Nucl. Phys.* **B 647** (2002) 344 [[hep-th/0206153](#)].

- [15] S.R. Coleman and K.-M. Lee, *Wormholes made without massless matter fields*, *Nucl. Phys.* **B 329** (1990) 387.
- [16] S.W. Hawking, *General relativity, an Einstein centenary survey*, S.W. Hawking and W. Israel eds., Cambridge University Press, Cambridge U.K. (1979).
- [17] G.W. Gibbons, M.B. Green and M.J. Perry, *Instantons and seven-branes in type IIB superstring theory*, *Phys. Lett.* **B 370** (1996) 37 [[hep-th/9511080](#)].
- [18] E. Bergshoeff, A. Collinucci, A. Ploegh, S. Vandoren and T. Van Riet, *Non-extremal D-instantons and the AdS/CFT correspondence*, *JHEP* **01** (2006) 061 [[hep-th/0510048](#)].
- [19] J.M. Maldacena, *The large- N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [[hep-th/9711200](#)].
- [20] N. Seiberg and E. Witten, *The D1/D5 system and singular CFT*, *JHEP* **04** (1999) 017 [[hep-th/9903224](#)].
- [21] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [[hep-th/9802109](#)].
- [22] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [[hep-th/9802150](#)].
- [23] V.A. Rubakov and O.Y. Shvedov, *A negative mode about euclidean wormhole*, *Phys. Lett.* **B 383** (1996) 258 [[gr-qc/9604038](#)].